The orbits of visual binary and multiple stars obtained by the Apparent Motion Parameters method during the last 40 years

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Abstract

Summed many years of work at Pulkovo, the orbits of 67 wide pairs of visual double and multiple stars (included in 64 systems) which were obtained by the Apparent Motion Parameters (AMP) method are presented. This short arc determination orbit method is based on the most reliable astrometric and astrophysical data corresponding to one instant of time. The rest of the observations accumulated in the world serve to control the quality of the orbit and refine some parameters. All early determined AMP-orbits were compared with new observations, some of them were recalculated, new ones were added. For the stars of Pulkovo program of observations with a 26-inch refractor, the Gaia DR2 data were analised. Based on these data, the orbits of 16 stars were calculated. In 20 cases from 67, the quasi-instant motion according to the Gaia DR2 data at the instant 2015.5 contradicts the motion according to all-world observations. A possible reason is the presence of inner subsystems. The orientation of the obtained orbits in the galactic coordinate system is also given.

Introduction

Double stars were first discovered by William Herschel at the end of the 18th century. In the 19th and 20th centuries, they were actively discovered by John Herschel, Wilhelm and Otto Struve, Paul Couteau and many other researchers. For stars with orbital periods of no more than 100-200 years, reliable orbits were determined, which made it possible to determine the masses of stars and derive fundamental laws relating the masses, spectra, luminosities and parallaxes of stars.

Wide, slowly circulating binary and multiple stars have been deprived of the attention of researchers, since from the moment of their discovery to the present time, observations cover a small arc of the orbit. However, such stars may turn out to be outer pairs of open or not yet discovered multiple systems. Namely, towards multiple systems, the components of which are being discovered more and more, the interest of researchers has now shifted - see works (Hale, 1994; Agati et al., 2015; Tokovinin, 2021). Their study ultimately carries information about star formation and the dynamic evolution of subsystems in our Galaxy. Therefore, the problem of determining the orbits of poorly studied wide binary stars remains relevant.

Observations of double stars have been a traditional theme of the Pulkovo Observatory since its opening in 1839. Therefore, when a 26-inch refractor appeared in Pulkovo after the Great Patriotic War, the priority task was to resume observations of wide binary stars in accordance with the capabilities of this telescope ($\rho > 3''$).

Especially for determining the orbits of such stars, the Apparent Motion Parameters method (AMP) was completed, see (Kiselev, Kiyaeva, 1980). This method was previously used to determine the orbit of an artificial satellite of the Earth from one photograph with many exposures (Kiselev, Bykov, 1973).

1 Application of the AMP-method

The AMP- method is designed to determine the initial orbits of wide visual binaries with a long period of revolution in position and velocity at one point in time based on the results of observations obtained by various available methods. These are the parameters of the apparent relative motion (AMPs) at one instant $T_\circ$: the distance between the components (ρ), the position angle (θ), the visible relative motion (μ) and the position angle direction of the visible motion (ψ), as well as the radius of curvature (ρ_c). The most accurate AMPs are obtained from homogeneous observations (basis) made with a single telescope to eliminate instrumental systematic errors (see, e.g., series of photographic observations with a 26-inch refractor of the GAO RAS for 1957–2007: (Kiselev et al., 2014; Izmailov et al., 2016).

In addition, the following data are necessary. They are: the parallax p_t (to relate linear and angular quantities), the relative radial velocity of the components $\Delta V_r = V_{rB} - V_{rA}$ (km/s), obtained from spectroscopic observations (to calculate the space velocity vector of the satellite relative to the main star), and an estimate of the sum of the masses of the components ΣM, according to the data on physical properties of these stars.

If it is possible to determine all five parameters, including the radius of curvature, then the distance between the components r in astronomical units can be calculated by the formula:

$$r^3 = k^2 \frac{\rho\rho_c}{\mu^2} |\sin(\theta - \psi)| \tag{1}$$

where $k^2 = 4\pi^2\Sigma M$ is dynamic constant if distance is measured in au, time in years, mass in $M_\odot$.

Then we get two position vectors that correspond to the position of the secondary component symmetrically with respect to the picture plane, and, consequently, two orbits ($\pm\beta$). Here β is the angle between the spatial position of the satellite and its projection on to the picture plane, which can be calculated by the formula:

$$\beta = \pm \arccos\left(\rho/(p_t r)\right) \tag{2}$$

Sometimes, in agreement with the entire series of observations (see *Washington Double Star Catalog* — WDS, Mason et al. (2016)) one can choose one solution.

If the radius of curvature cannot be determined, then the distance between the components, we obtain according to the condition necessary for an elliptical orbit:

$$\frac{\rho}{p_t} \le r < \frac{2k^2}{v^2}, \tag{3}$$

where v is the modulus of spatial velocity in au/year. In this case, we get a family of orbits. Each orbit of the family is characterized by the angle β.

If it is impossible to estimate the radius of curvature from a short arc of homogeneous observations, but the entire series of available observations reflects a nonlinear elliptical motion, then a more correct orbit can also be chosen from the family in agreement with the entire series of observations.

In order to obtain high-accuracy AMPs, at the Pulkovo Observatory much attention was paid to obtaining a homogeneous long-term series, from which the relative motion is determined more confidently. However, for some stars it was necessary to break the homogeneity and use additional observations from the WDS catalog.

The AMP orbits for the two stars included in this work (HIP 12706 and HIP 33287) required minor corrections and were refined using the differential correction method using Tokovinin's ORBITX program (Tokovinin, 1992). For these stars, Table 2 compares the AMPs obtained using the basis indicated in the table and the AMP ephemeris corresponding to the orbits refined using the ORBITX program.

Sufficiently accurate radial velocities are not available in the literature for all the stars in our program. Sometimes this parameter also had to be selected in agreement with the observations. Then the longitude of the ascending node and the longitude of the periastron from the node are determined with an accuracy of up to 180° and the ephemeris of the orbits in the projection onto the picture plane coincide. The remaining parameters remain the same, but the orientation of the orbital plane in the Galaxy changes. Thus, for the stars presented in this paper, we obtain 1, 2, 4 solutions or a family of orbits.

For the initial estimation of the sum of the masses of the components, we used the "mass-spectrum-luminosity" ratio according to the handbook (Allen, 1999) and data from the WDS catalog. However, there are no spectral types for many stars in the WDS catalog. Therefore one had to use different sources, which do not always give an unambiguous result. At present, the best way to get mass estimates is to use evolutionary track diagrams that link color index and magnitude (Girardi et al., 2000; Bressan et al., 2012).

Since most of the components of visual binaries in our program turned out to be spectroscopic binaries, data on them are contained in the Tokovinin *Catalog of Multiple Stars* — MSC, (Tokovinin, 2018). Nevertheless, in some cases, our studies of dynamics lead to an excess of masses in stellar systems, which should be taken into account in further studies.

The more accurate the input data, the more reliable the orbit. We determine the errors of each orbital parameter by the total influence of the errors of all initial parameters. Since this effect is asymmetric with respect to the obtained solution, we present two errors that correspond to the maximum and minimum values of the given orbital element. The sum of the masses of the components is both the initial and the refined parameter, it is functionally related to parallax, so we fix it.

The main advantage of this method is that it is possible to control the quality of the solution obtained, because to determine the apparent movement parameters, we do not use all available observations, but only the basis. You can also check the consistency of the source data with each other. This fundamentally distinguishes it from methods where the main criterion is agreement with positional observations, which always make it possible to obtain an orbit, but work poorly on a short arc.

Naturally, the AMP-method has limitations, and its application requires an individual ap-

proach. First of all, the impossibility of reconciling all the initial data is due to the fact that the internal subsystems can distort the AMPs, as well as the accepted stellar masses. In this case, the method must not be applied.

In those cases when it is possible to obtain an orbit by the AMP- method, it is currently more correct than other orbits, since it is based on a whole complex of observation results of both an astrometric and astrophysical nature.

2 Results

The purpose of this work is to summarize many years of work, to systematize the results obtained earlier and to add new orbits.

We revised all orbits obtained over 40 years, compared its with modern observations from the WDS catalog, with CCD observations on a 26-inch refractor in Pulkovo for 2003-2019 (Izmailov et al., 2010; Izmailov, Roshchina, 2016; Izmailov et al., 2020) and with the relative positions and motions calculated by us from high-precision data from the Gaia DR2 (Gaia Collaboration et al., 2018).

Most of the objects of our study have separations from $3''$ to $39''$, except for a few closer binaries that are not included in the Pulkovo Observatory observational program, and wider pairs. Basically, these are dwarfs of spectral types F, G and K from the nearest vicinity of the Sun. A total of 67 pairs included in 64 visual binary and multiple systems were considered. Among them are three visual triples (ADS 48 ABF, ADS 7034 ABC, and ADS 10288 ABC), for which we have determined both inner and outer orbits. An unambiguous AMP-solution was obtained for 33 pairs, 2 solutions for 14 pairs, 4 solutions for 3 pairs, families of AMP-orbits were calculated for 17 pairs.

For 29 stars, the orbits have been improved. Most often, the improvement consisted in using a more accurate parallax from the Gaia DR2 catalog and a homogeneous series of Pulkovo CCD observations instead of heterogeneous ones. In this paper, we also publish the orbits ADS 895 and HIP 12706 obtained for the first time. Orbits of 16 stars were completely recalculated from the Gaia DR2 data (positions, proper motions, parallaxes and radial velocities), and the first orbits of seven stars were obtained.

In 20 out of 67 cases, the quasi-instantaneous motion according to the Gaia DR2 data contradicts the average motion according to all-world observations. This is grounds for suspecting the presence of an additional satellite. Therefore to improve our orbits we used only Gaia DR2 parallaxes, new data from the literature, and elongated Pulkovo series. See, for example, (Shakht et al., 2020; Romanenko, Izmailov, 2021; Kiyaeva et al., 2021).

On the other hand, 24 pairs are known to have confirmed inner subsystems. Usually, short-period spectral satellites did not affect the AMP-values, and only increased their errors. The satellite's motion was taken into account when calculating the outer orbit, where it possible.

The results are presented on the website izmccd.puldb in the form of spreadsheets and an Appendix, which includes comments on each star and graphs illustrating the comparison of ephemeris with observations, as well as the region of stable solutions for families.

Here we give only Table 1, which presents data characterizing the components of the stellar pairs studied, series of observations and obtained solutions for each star.

Designations: Columns 1-3 give numbers of pairs (1 — in order, 2 — according to the Hipparcos catalog (ESA SP-1200, 1997), 3 — according to the ADS catalog (Aitken, Doolittle,

Table 1: Identifiers, WDS data on the studied stars, characteristics of the series of observations and the obtained results

N	HIP	ADS	Comp.	WDS	m_1	m_2	$Sp1_W$	$Sp2_W$	$Sp1_T$	$Sp2_T$	$T1_W$	$T2_W$	n_W	$T1_P$	$T2_P$	n_P	n_C	k_0	NOTE	Ref.
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	50	-		00006-5306	6.55	9.85	G0IV	-	-	-	1836	2015	27	-	-	-	-	4	C	[1]
2	473	48	AB	00057+4549	8.98	9.15	K6	M0	K6V	K6V	1876	2015	399	1961	2019	148	056	2	CVG	[2,3]
3	473/428	48	AB-F	00057+4549	8.98	10.19	K6V	M2e	K6V	K8V	1897	2007	37	1968	1995	117	000	F	VWM	[2]
4	1475	246	AB	00184+4401	8.13	11.04	M1V	M3.5V	K8V	M0V	1860	2015	123	1994	2019	007	160	1	V(C)	[4]
5	2844	497	AB	00360+2959	8.39	9.05	G2V	G7V	G7V	G6V	1824	2015	134	1971	2019	087	066	1	MVWN̂	[0]
6	5110	895		01055+1523	9.24	9.93	K0		K2.5V	K7V	1829	2007	98	1960	2019	046	093	2	V(C)WM	[0]
7	12706	-	AB	02433+0314	3.54	6.18	A1V		-	-	1825	2014	235	-	-	-	-	2	CO(M)W	[0]
8	12777	2081	AB	02442+4914	3.89	9.14	F7V	M1.5	F6V	K8V	1782	2013	76	-	-	-	-	1	VWG	[0]
9	12780/779	2098	AB	02442-2530	7.5	8.1	G3V		G2V	K0V	1835	2013	29	-	-	-	-	F	VWM	[1]
10	15058	2416		03140+0044	8.14	8.17	F8		G9V	K0V	1831	2012	167	-	-	-	-	2	C(M)	[1]
11	15220	2427	AB	03162+5810	10.30	11.38	M2V		M0V	M0V	1914	2013	82	1971	2019	100	149	1	V(C)G	[0]
12	17129	2668	A-Bb	03401+3407	7.52	7.60	F9V		F9V	F4V	1823	2012	230	2003	2019	000	045	2	(C)MW	[5]
13	17666	2757	AB	03470+4126	8.20	8.82	K1V	K2V	G8V	K1V	1822	2011	129	1960	2019	099	058	1	VWM	[4]
14	20390	TTau	NS	04220+1932	5.5	8.1	G5V		-	-	1990	2002	69	-	-	-	-	4	CWM	[0]
15	32609	5436	AB	06482+5542	6.28	6.34	dF5	dF6	F6V	F5V	1821	2014	126	1962	2019	033	082	2	MVW	[0]
16	33287	5570		06555+3010	8.72	8.97	G0		K0V	K0V	1831	2015	110	-	-	-	-	2	C(M)WO	[1]
17	35550	5983		07201+2159	3.55	8.18	A9III	K3V	(F6V)	K1V	1822	2013	254	1972	2019	108	074	1	MCVW	[6]
18	40527/32	6646	AB	08165+7930	8.40	8.64	G0		F3V	F8V	1832	2005	68	1962	2003	041	000	F	MVW	[0]
19	41184/81	6783		08243+4457	7.79	9.39	G0		G2V	K1V	1830	2012	41	1996	2006	017	000	F	VWG	[0]
20	43426	7034	AB	08508+3504	7.41	7.48	F8		G0V	F9V	1821	2012	125	1962	2019	020	066	1	VWG	[7]
21	43426	7034	AB-C	08508+3504	7.41	11.69	F8		G0V	K6V	1941	2005	4	-	-	-	-	F	VWMG	[7]
22	45343/*	7251	AB	09144+5241	7.79	7.88	M0V	M0V	K7V	K7V	1821	2015	449	1963	2019	185	075	1	CVW	[8]
23	48429	7551		09524+2659	9.12	9.50	K0		G7V	G9V	1830	2014	134	1972	2019	009	120	1	(C)WG	[0]
24	48804	7588		09572+4554	8.89	9.75	G0		G0V	G6V	1828	2013	46	1971	2019	011	107	1	VWG	[7]
25	50583	7724	AB	10200+1950	2.37	3.64	K0III		-	-	1782	2015	834	1992	2008	020	020	2	VW(C)	[9]
26	-	8002		10596+2527	8.57	9.22	K2	K5	K3V	K4V	1899	2014	102	1970	2018	061	056	1	CV	[10]
27	54407	8065		11080+5249	7.65	9.03	F8V		F8V	G7V	1830	2010	74	1970	2018	024	053	1	VWG(C)	[0]
28	54952	8100	AC	11152+7329	7.77	11.34	K5		K3V	M0.5V	1858	2011	66	1969	1999	041	000	2	(C)W	[11]
29	56622	8236		11366+5608	7.73	8.17	G5	K7V	G2V	G7V	1828	2007	116	1962	2019	061	079	1	MVW(C)	[5]
30	56809	8250	AB	11387+4507	6.53	8.23	G0V		G0V	K2V	1782	2015	137	1969	2019	048	107	1	MVWC	[10]
31	60831/32	8561		12281+4448	7.49	8.08	F9V		F8V	G3V	1791	2012	99	1971	2007	025	008	2	VWG	[7]
32	62561	8682	AB	12492+8325	5.29	5.74	A1IIIsh		-	-	1820	2011	68	1969	2007	023	000	F	MVW	[0]
33	64405	8814		13120+3205	7.40	7.64	F6V		F4V	F3V	1843	2014	255	1985	2004	032	000	2	VW(C)G	[0]
34	65011/12	8861	AB	13196+3507	9.62	11.90	M0.5V	M3V	K7V	K9V	1827	2012	53	1971	2019	081	215	2	(C)MW	[12]
35	66195	8959		13341+6746	9.26	9.56	G1V		G3V	G5V	1832	2007	47	1971	2019	051	137	1	VW(C)G	[0]
36	67422	9031		13491+2659	7.36	8.15	K4V	K6V	K3V	K5V	1823	2015	875	1962	2019	097	150	1	CG	[0]
37	67871	9048		13540+3249	8.63	8.97	F8		F9V	G0V	1823	2015	66	1962	2005	034	000	1	VWG	[7]
38	68588	9090		14024+4620	10.05	10.26	M2	M2.5	K8V	K7V	1889	2015	112	1962	2018	077	066	1	V(C)G	[0]
39	–	-		14051+4913	11.80	11.98	K4/5		G7V	G7V	1902	2016	18	1969	1975	022	000	F	VWG	[7]
40	69442	9167		14131+5520	9.06	9.42	K2		K1V	K1.5V	1831	2013	202	1971	2019	073	073	1	M(C)VW	[0]
41	69751	9192		14165+2007	6.47	8.42	F6V		F5V	G7V	1830	2015	157	2003	2019	000	119	1	CVW(M)	[1]
42	71782	9346	AB	14410+5757	7.53	8.32	K0IV	G5IV	G9V	G4V	1830	2015	97	1979	2019	036	078	1	(M)VW	[18]
43	71876	9357		14421+6116	6.33	9.16	F4V		F4V	K1V	1832	2015	46	2004	2019	000	040	1	VWG(C)	[7]
44	73846	9497	AB	15055-0701	8.09	8.76	G0		F6V	G1V	1873	2014	83	-	-	-	-	1	CW(M)	[1]
45	74666/74	9559	AB	15155+3319	3.56	7.89	G8III		(K3V)	G1V	1780	2015	109	1994	2005	010	000	F	VWG	[0]
46	75809/29	9696	AB	15292+8027	6.64	7.30	G0IV-V		(G3V)	G8V	1823	2010	87	1969	2004	044	000	F	VWG	[0]
47	80349	10044		16242+3702	8.43	8.79	K0		K1V	G3V	1823	2009	99	1962	2019	026	255	F	MVW	[0]
48	83020	10288	AB	16579+4722	7.93	10.85	K0		K2V	M0V	1908	2007	33	1993	2019	010	150	1	V(C)G	[0]
49	83020/06	10288	AB-C	16579+4722	7.93	8.05	K0V		K2V	K1.5V	1823	2011	38	1993	2019	011	000	F	MVWG	[0]
50	83451/54	10329		17033+5935	8.76	10.34	K4V		K2V	K7V	1830	2006	54	1970	2019	022	050	1	VWG	[0]
51	83608	10345	AB	17053+5428	5.66	5.69	F7V		F5V	F5V	1779	2015	794	1965	2019	019	150	1	CW	[12]
52	83988/96	10386	AB	17102+5430	8.85	9.21	K6V	K6V	K4V	K6V	1830	2012	42	1961	2019	027	033	1	VW	[4]
53	86614/20	10759	AB	17419+7209	4.60	5.59	F5IV	F8V	F5V	F7V	1800	2015	172	1980	2005	076	003	F	MVW	[0]
54	88136/27	11061	AB	18002+8000	5.70	6.00	F7V	F7V	F7V	F6V	1782	2014	146	1964	2006	080	002	F	MVW	[0]
55	91768/72	11632	AB	18428+5938	9.11	9.96	M4	M4.5	K9V	K9V	1831	2015	608	1961	2019	189	158	1	CV	[13]
56	93873/99	GL745		19072+2053	10.95	10.99	M2V	M2V	K9V	K9V	1897	2012	17	1996	2007	009	000	F	VWG	[0]
57	94336	12169	AB	19121+4951	6.54	6.67	G3V	G3V	G2V	G3V	1819	2015	286	1961	2019	096	157	1	VW	[4]
58	96895/901	12815	AB	19418+5032	6.00	6.23	G1.5V		G2V	G2V	1800	2014	582	1960	2007	162	000	F	VWGM	[0]
59	97222	12889	AB	19456+3336	8.47	8.58	K3V		K1.5V	K1.5V	1828	2015	450	1995	2019	008	103	1	CV	[14]
60	97295	12913	AB	19464+3344	5.06	9.25	F5V		F6V	K2V	1822	2014	138	1995	2019	009	062	2	VW	[14]
61	97292	GL767	AB	19464+3201	10.38	11.15	M0.5V	M2V	K8V	M0V	1935	2015	81	1971	2019	035	311	1	(M)V(C)	[15]
62	104214/17	14636	AB	21069+3845	5.20	6.05	K5V	K7V	K5V	K6V	1753	2015	1672	1958	2019	328	254	1	CV	[16]
63	-	14878	AB	21200+5259	7.71	7.87	F8V		G0V	G8V	1828	2015	100	1985	2005	025	000	F	VW	[10]
64	105502	14909	AB	21221+1948	4.20	9.3	K0.5III		-	K0V	1780	2013	75	1994	2005	009	000	F	MVW	[0]
65	108456/61	15571	AB	21582+8252	7.00	7.47	F6IV-V		(F8V)	G6V	1825	2014	157	1960	2003	102	000	2	CMVW	[17]
66	108917	15600	Aa-B	22038+6438	4.45	6.40	A3m		(F3V)	F7V	1779	2015	276	1963	2019	089	149	4	MW	[0]
67	118281	17149	AB	23595+3343	6.46	6.72	F8V		F8V	F8V	1777	2015	606	1980	2017	011	047	1	V(C)WG	[0]

Note: * — ADS 7251 B = HIP 120005. Designations are given in the text. The last column contains a link to the article in which this orbit was obtained ([0] — if in this article; [1]= (Kiyaeva et al., 2017); [2]= (Kiyaeva et al., 2020); [3]= (Kiyaeva et al., 2001); [4]= (Romanenko, Izmailov, 2021); [5]= (Kiyaeva, Izmailov, 2018); [6]= (Shakht et al., 2007); [7]= (Kiyaeva, Romanenko, 2020); [8]= (Shakht et al., 2020); [9]= (Romanenko, Kiselev, 2014); [10]= (Romanenko, 2018); [11]= (Grosheva, 2006b); [12]= (Kiselev et al., 2000); [13]= (Kiselev et al., 2009); [14]= (Romanenko, 2017); [15]= (Kiyaeva, Gorynya, 2015); [16]= (Shakht et al., 2017); [17]= (Grosheva, 2006a); [18] = (Kiyaeva et al., 2021)).

1932)), column 4 gives the components designations, 5 is the WDS number (Mason et al., 2016), 6–9 are the magnitudes and spectral types of the components according to WDS, 10 and 11 are the spectral types of the components estimated by us according to the effective temperature Teff from the Gaia DR2 data (Gaia Collaboration et al., 2018) and monograph (Agekian, 1981) under the assumption that this component is a dwarf (otherwise, the result is given in parentheses); 12 and 13 are the beginning and end of all WDS observations (version of 2016), 14 is the total number of observations in the WDS, 15 and 16 are the beginning and end of Pulkovo observations, 17 is the number of Pulkovo photographic observations, 18 is the number of Pulkovo CCD observations, 19 is the number of AMP-orbits (k0) or F — family, 20 is code of one letter each: M is the presence of an inner subsystem (in brackets, if the satellite is assumed by us, but not yet confirmed), V is the presence of the radial velocity from observations, C is the presence of the radius of curvature from the observations of the basis (in parentheses, if over the entire arc, but not determined from the short basis), W is a wide pair ($a > 100$ au), O is AMP-orbit improved by the ORBITX program (Tokovinin, 1992), G is AMPs of the final chosen orbit calculated from observations and proper motions of Gaia DR2; column 21 is a reference to the article in which this orbit was obtained.

Note that photographic observations with the 26-inch refractor at Pulkovo ended in 2007 and CCD observations began in 1996, but in this work we have used only systematic CCD observations since 2003 to 2019.

Table 2 presents the initial data for obtaining the AMP-orbit. Namely: the apparent motion parameters and additional parameters, as well as their errors.

Table 3 lists the orbital elements for 50 pairs, their errors depending on the initial parameters, as well as the orientation of the obtained orbits in the galactic coordinate system and the average-weight values (O-C), corresponding to the entire series and to the Gaia DR2 observation. The assignment of weights to observations is explained in the article (Kiyaeva et al., 2017). The initial data errors correspond to 1σ, then the orbital elements fall into the specified error range with a probability of 68%.

Table 4 shows the orbital elements for 17 pairs (families) corresponding to singular points, i.e. having a minimum period ($\beta = 0°$), a minimum eccentricity, and limiting periods for reliable orbits of the families.

Here we should note the peculiarity inherent in the algorithm for determining orbits by position and velocity. See the monographs (Subbotin, 1968) and (Kholshevnikov, Titov, 2007).

First of all, we obtain the semi-major axis of the orbit a according to the energy integral by the formula:

$$\frac{1}{a} = \frac{2}{r} - \frac{v^2}{k^2} \tag{4}$$

For values of β close to β_{max}, when the orbit is close to parabolic, the error in determining a is large. Then calculating the ephemeris from the found orbit, we get an erroneous value of r:

$$r = a(1 - e\cos E) \tag{5}$$

Here e is the eccentricity, E is the eccentric anomaly. In addition to the inaccurate value of a, the value of r is affected by the position that the satellite occupies in orbit at the instant T_0. It is obvious that a greater uncertainty is obtained near the periastron. In Table 4, we include only reliable solutions when the cycle of calculating the orbit and its ephemeris is closed and all

the orbits of the family fit the observed series well.

In the Appendix (on the site izmccd.puldb) comments and graphics are given for each star. The comments briefly describe the history of the study of the system and the rationale for this result. The graphs show series of observations, orbit ephemeris, and the direction of motion according to Gaia DR2 data at the instant of 2015.5.

The following dependences are presented: $\rho(t)$, $\theta(t)$ and $y(x)$. A graph in the picture plane $y(x)$ is sometimes presented in two forms: a fragment of an arc covered by observations, and a complete orbit during the entire period. Then you can see how small the observed arc is.

For all families, the functional dependence $lg(a) = f(e)$ is additionally presented as a graph and the region of possible influence of the Galaxy is determined. For visual triple stars (ADS 48, ADS 7034 and ADS 10288), the stability region is also determined as it was done in the work (Kiyaeva, Romanenko, 2020).

3 Conclusion

The orbits of 67 wide pairs of visual binary and multiple stars included in 64 systems are presented. Orbits were unambiguously obtained for 33 pairs, among them 22 pairs have a semi-major axis of the orbit of more than 100 au. and periods from 600 to 6000 years. For such wide pairs, the AMP- method, in contrast to other methods, makes it possible to obtain more reliable orbits.

For 40 years, each researcher from the group of A.A.Kiselev has being developed his own algorithms for determining the AMP-orbits, corresponding to the features of the objects of interest to him, depending on the available initial data. To achieve research uniformity and ease of use, we have created a single program that, using the data in Table 2 for each pair, calculates orbit elements with errors, orientation, ephemeris, and deviations to observations (Tables 3 and 4, as well as data for graphs). This program can serve as a tool for determining the AMP-orbits of yet unexplored visual binaries of the Pulkovo program as the missing data become available.

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